

Today: Periodic functions, in particular sine, cosine, tan, —

Periodic Phenomena: Ex: heartbeat, sunset/sunrise, seasons

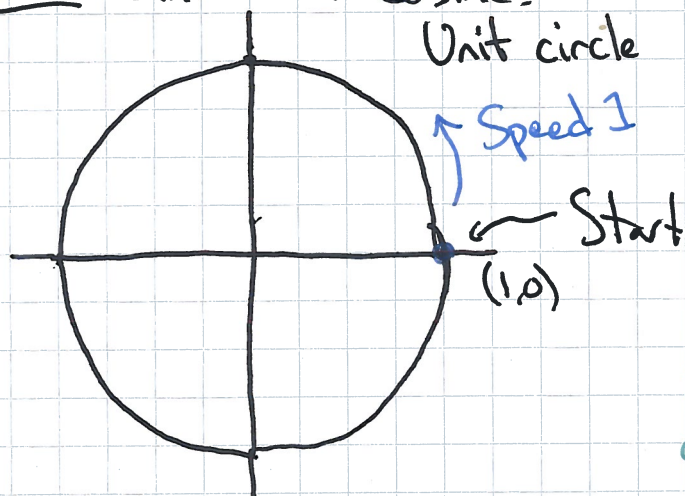
Period = ~ 1 sec $\quad \quad \quad \approx 1$ day $\quad \quad \quad = 1$ year

the length of time until it repeats.

Molecular Vibrations: $\sim 10^{-13}$ or 10^{-14} sec.

Precession of Earth's axis: $\sim 26,000$ years

Ex: sine and cosine.



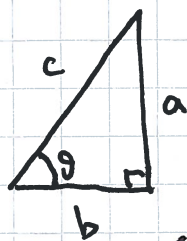
If you're traveling counterclockwise on the unit circle, speed of 1.

Then your position is 2π -periodic

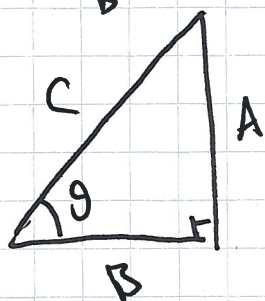
$\sin(t) = y\text{-coord at time } t$
 $\cos(t) = x\text{-coord at time } t$

Trigonometry

if $\theta < 90^\circ$

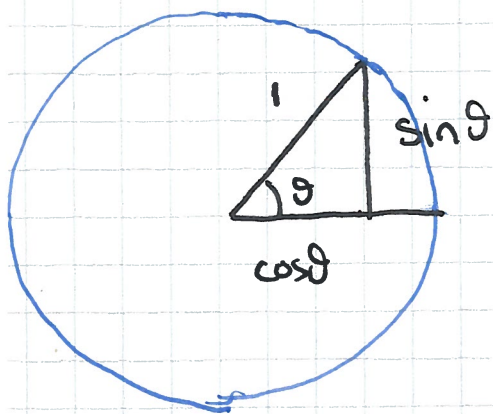


$$\sin \theta = \frac{a}{c} \quad \cos \theta = \frac{b}{c}$$

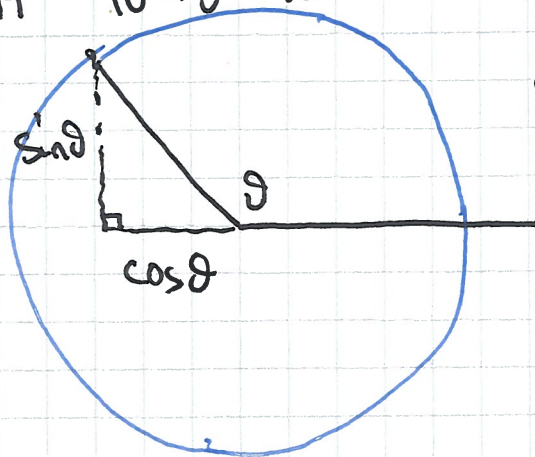


$$\frac{a}{c} = \frac{A}{C} \quad \frac{b}{c} = \frac{B}{C}$$





if $90^\circ < \theta < 180^\circ$



$\sin \theta > 0$

$\cos \theta < 0$

$$\cos \theta = \sin(\theta + \underbrace{90^\circ}_{\text{Degrees}}) \quad (\text{or } \cos(\underbrace{t}_{\text{Radians}}) = \sin(\underbrace{t + \frac{\pi}{2}}_{\text{Radians}}))$$

Degrees

One full revolution = 360°

$\frac{1}{2}$ a rev = 180°

$\perp$ = 90°

$\approx 57^\circ$

- Historical relic of Babylonians (base 12)



Radians

2π

π

$\pi/2$

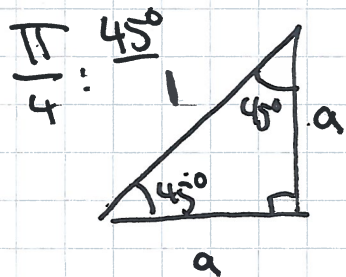
r means arc length is r

- Derivatives work better
- Everything is better

Special values of sine & cosine:

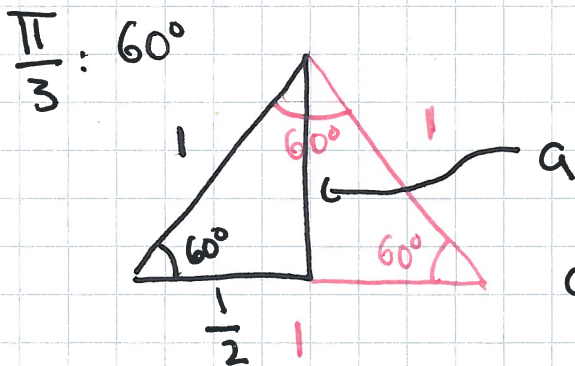
$$\sin\left(\frac{\pi}{2}\right) = 1 \quad \text{Max value}$$

$$\sin\left(\frac{3\pi}{2}\right) = -1 \quad \text{Min value}$$



$$\begin{aligned} a^2 + a^2 &= 1 \\ a^2 &= \frac{1}{2} \\ a &= \frac{1}{\sqrt{2}} \end{aligned}$$

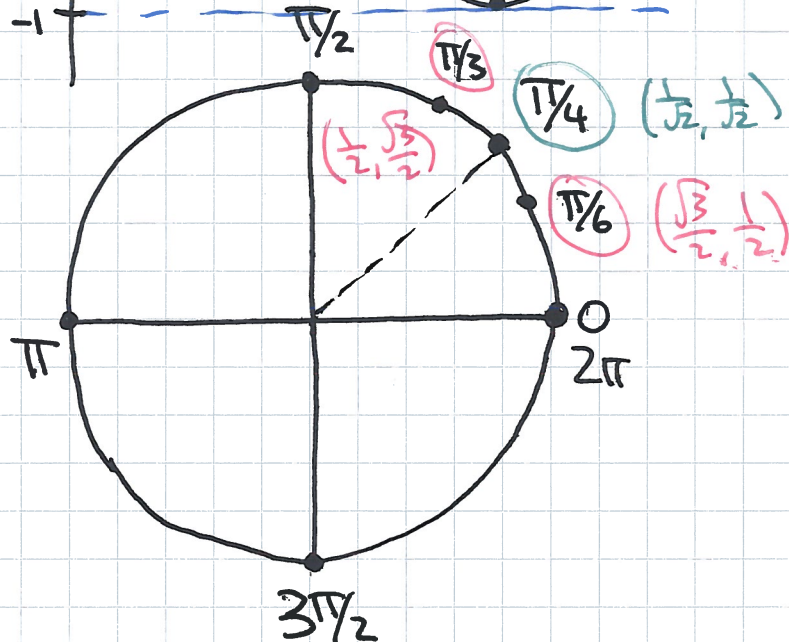
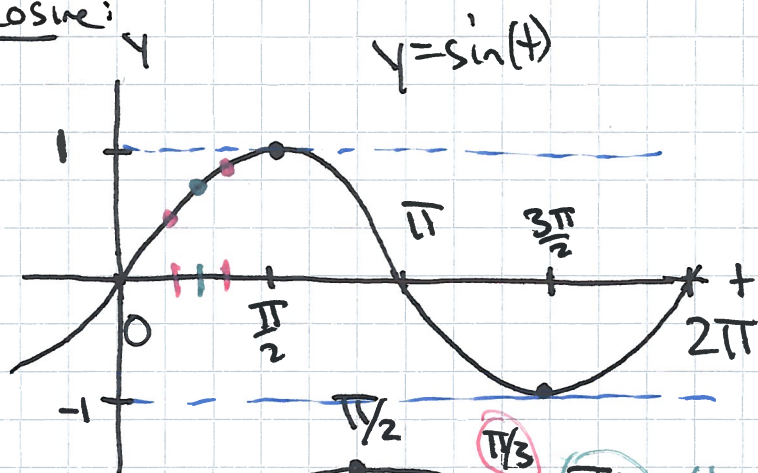
$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$



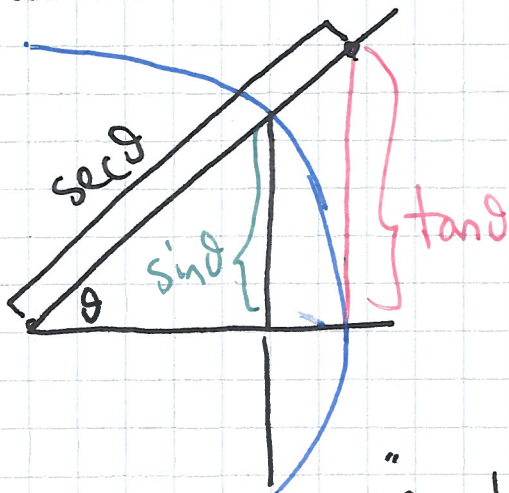
$$a^2 + \frac{1}{4} = 1$$

$$a = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$



Radius!



jiva: "bowstring"

sinus: bend
sine

tangens: touching
tangent

"complementi"
co

secans: "to cut"
secant

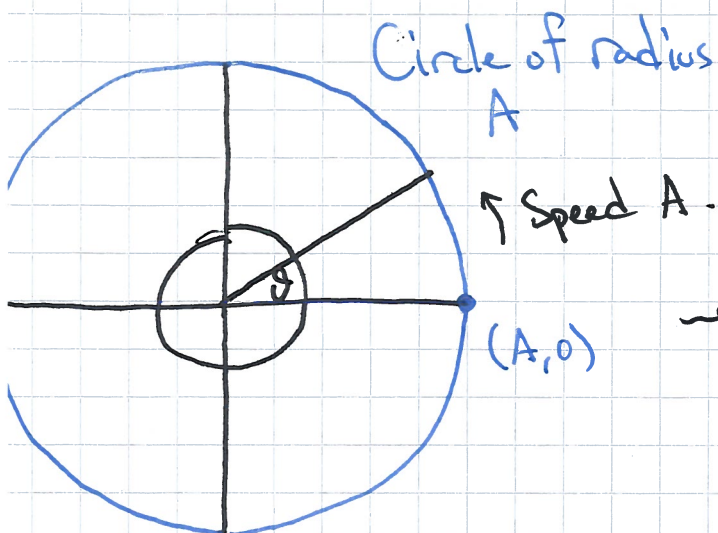
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

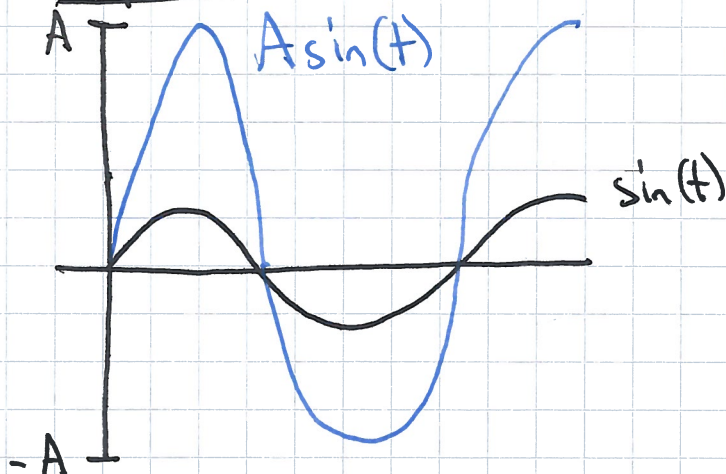
$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Amplitude, Frequency, Phase:



Amplitude: Different sized circle

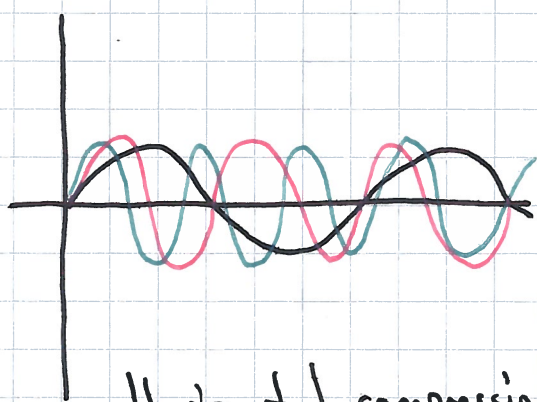
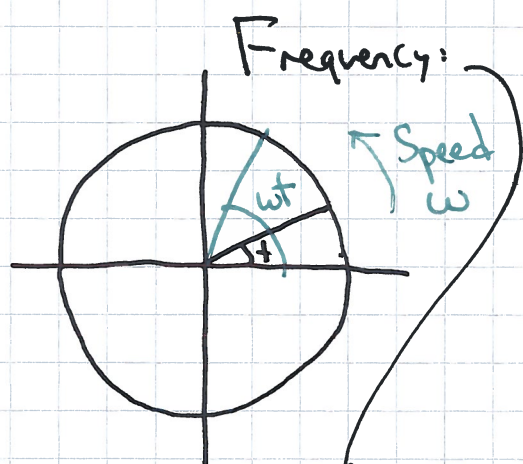


Vertical dilation by A .

Amplitude = size of the wave vertically

Ex: sound waves

→ Double: ↑ one octave.
freq



Horizontal compression

going at speed w instead of speed 1.

Phase: start at a different point

